

PREDICTIONS OF THE TOP MASS IN MINIMAL SUPERSYMMETRIC LEFT-RIGHT MODEL

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Abstract

The one-loop evolution of Yukawa couplings in the minimal supersymmetric left-right model (MSUSYLR) model with a wide variation of the right handed breaking scale M_R from 1 TeV to 10^{18} GeV is studied assuming that all third generation Yukawa couplings are equal and in the fixed point domain of the top quark Yukawa coupling (h_t) at the Plank scale. We show that: (1) The top quark Yukawa coupling h_t displays a fixed point behaviour that is similar to that of the minimal supersymmetric standard model (MSSM). (2) The MSUSYLR model predicts a value of the top mass in the interval 177 to 184 GeV for α_s in the interval 0.11 to 0.12. (3) A large value of $\tan\beta$ is required to reproduce the correct mass of the bottom quark and tau lepton. (4) With the experimental value of the ratio $\frac{m_b(m_b)}{m_\tau(m_\tau)}$ as an input the range of the right handed symmetry breaking scale M_R can be predicted. (5) The numerical value of the Majorana Yukawa coupling h_M can be calculated which is otherwise a completely free parameter.

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There is a lot of interest among physicists in the possible measurement of the top quark mass m_t by the CDF group in the vicinity of 174 GeV [1]. What does this kind of a value mean for various theoretical models trying to generate m_t ? It is well-known that [2] the top quark mass m_t may get fixed at an infrared stable fixed point by the low energy structure of the renormalization group equations (RGE) of the corresponding Yukawa coupling h_t . These equations determine the evolution of h_t from a large mass scale ($M_X \simeq 10^{19} \text{GeV}$) to m_t . One obtains a universal value of $h_t(m_t)$ for a large domain of values of $h_t(M_X)$. This result is very interesting in that it shows how the details of the possibly complicated symmetry breaking mechanisms at M_X might be obliterated by the renormalization group equations, whose fixed point structure emerges dominant at low energies. The insensitivity to the ultraviolet behaviour is a hallmark of infrared stable fixed points in all branches of physics. In this communication we want to test this insensitivity by studying the behaviour of the top Yukawa coupling in the Minimal Supersymmetric Left Right Model (MSUSYLR) in comparison with that in the Minimal Supersymmetric Standard Model (MSSM).

The MSSM admits an $N=1$ global supersymmetry by construction and consequently the spectrum includes the superpartners of all the fermions and bosons of the Standard Model (SM), extended to two Higgs doublets. R-parity; $R_p = (-)^{3B+L+2S}$ (with B,L,S as baryon number, lepton number and spin respectively) distinguishes between particles ($R_p = 1$) and superparticles ($R_p = -1$). The R_p violating terms, if present in the superpotential, lead to lepton and/or baryon non-conservation. Unless one of these two conservations breakdowns is very small in magnitude, they will induce catastrophic proton decay unobserved in nature. The popular assumption in MSSM [3] has been to have the R_p -conservation built in by fiat though the R_p violating terms are allowed by gauge invariance and supersymmetry [4].

Sometime ago, it was noted that, when MSSM is extended to MSUSYLR, the unwanted R_p violating terms automatically vanish [5]. At the level of an underlying $SO(10)$ GUT, this can be easily understood. $SO(10)$ does not allow a singlet in the product representation $16 \times 16 \times 16$. This is a strong motivation to study the MSUSYLR model. Of course, spontaneous R_p breaking is allowed in the MSUSYLR model, however, being spontaneous in nature, this violation can be kept under desirable control at low energy.

Recently, a number of studies have been made of the top quark Yukawa coupling in the Minimal Supersymmetric Standard Model (MSSM) [6]. Here we perform a similar study for the SUSYLR model. The respective top and bottom Yukawa couplings h_t and h_b of the MSSM get embedded in the quark Yukawa coupling h_q of the MSUSYLR model at the scale M_R . Similarly, the tau lepton Yukawa coupling h_τ gets embedded in the lepton Yukawa coupling h_l . The symmetry breaking chain is as shown below.

$$\begin{aligned}
& MSUSYLR &= SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
M_R \implies & MSSM &= SU(3) \times SU(2)_L \times U(1)_Y \\
M_{SUSY} \implies & SM &= SU(3) \times SU(2)_L \times U(1)_Y \\
M_Z \implies & QED + QCD &= SU(3)_c \times U(1)_{em}
\end{aligned} \tag{1}$$

There are various scenarios of gauge coupling unification in the SUSYLR model. With the minimal choice of the Higgs fields, the right handed symmetry breaking scale has to be

comparable to the unification scale in order to get a consistent gauge coupling unification. However, one can enlarge the Higgs choice [7] or include the effect of the higher dimensional operators [8] to get a low energy right handed symmetry breaking scale. In this paper we stick to the minimal Higgs choice [9] and do the calculation in such a way that the results become independent of the specific model of gauge coupling unification. The right handed $SU(2)_R$ group is broken by the VEV of the scalar $\overline{\Delta}_2 = (1, 1, 3, \sqrt{\frac{3}{2}})$ under the SUSYLR symmetry group. The field $\Delta_1 = (1, 3, 1, \sqrt{\frac{3}{2}})$ has to be present to keep the left-right parity ($g_l = g_R$) intact. The scalar field $\phi = (1, 2, 2, 0)$ has the two MSSM Higgs doublets H_1 and H_2 embedded in it. The matter superfield representations of the MSUSYLR model and the corresponding representation in MSSM are given in tables 1 and 2. At the right handed symmetry breaking scale, the $U(1)_Y$ hypercharge emerges as a combination of the diagonal generator of $SU(2)_R$ and the generator of $U(1)_{B-L}$:

$$Y = \sqrt{\frac{3}{5}} T_R^3 + \sqrt{\frac{2}{5}} (B - L) \quad (2)$$

The matter superfields of the MSUSYLR model embed those of the MSSM in the following way. Thus Q_1 of the MSUSYLR contains the Q of the MSSM [see table 1 and table 2] while \overline{Q}_2 of the MSUSYLR contains the \overline{U} and \overline{D} of the MSSM. L_1 of the MSUSYLR contains L of the MSSM while \overline{L}_2 of the contains \overline{E} . The right handed neutrino gets a large Majorana mass at the right handed symmetry breaking scale.

<i>Superfield</i>	$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$	<i>Anomalous Dimension</i>
Q_1	$(3, 2, 1, \frac{1}{6}\sqrt{\frac{3}{2}})$	$\frac{1}{16\pi^2} [2h_Q^2 - \frac{8}{3}g_c^2 - \frac{3}{2}g_L^2 - \frac{1}{12}g_{B-L}^2]$
\overline{Q}_2	$(\overline{3}, 1, 2, -\frac{1}{6}\sqrt{\frac{3}{2}})$	$\frac{1}{16\pi^2} [2h_Q^2 - \frac{8}{3}g_c^2 - \frac{3}{2}g_R^2 - \frac{1}{12}g_{B-L}^2]$
L_1	$(1, 2, 1, -\frac{1}{2}\sqrt{\frac{3}{2}})$	$\frac{1}{16\pi^2} [2h_L^2 + 2h_M^2 - \frac{3}{2}g_L^2 - \frac{3}{4}g_{B-L}^2]$
\overline{L}_2	$(1, 1, 2, \frac{1}{2}\sqrt{\frac{3}{2}})$	$\frac{1}{16\pi^2} [2h_L^2 + 2h_M^2 - \frac{3}{2}g_R^2 - \frac{3}{4}g_{B-L}^2]$
Δ_1	$(1, 3, 1, \sqrt{\frac{3}{2}})$	$\frac{1}{16\pi^2} [h_M^2 - 4g_L^2 - 3g_{B-L}^2]$
$\overline{\Delta}_2$	$(1, 1, 3, -\sqrt{\frac{3}{2}})$	$\frac{1}{16\pi^2} [h_M^2 - 4g_R^2 - 3g_{B-L}^2]$
ϕ	$(1, 2, 2, 0)$	$\frac{1}{16\pi^2} [3h_Q^2 + h_l^2 - \frac{3}{2}g_L^2 - \frac{3}{2}g_R^2]$

Table 1: The Superfields in the MSUSYLR model. Representations and the anomalous dimensions

The Lagrangian density of MSSM in standard superfield notation is given by,

$$\mathcal{L} = h_\tau [L H_1 \overline{E}]_F + h_b [Q H_1 \overline{D}]_F + h_t [Q H_2 \overline{U}]_F, \quad (3)$$

while that of the MSUSYLR is given by [10]

$$\mathcal{L} = h_Q [Q_1^T \tau_2 \phi \tau_2 \overline{Q}_2]_F + h_l [L_1^T \tau_2 \phi \tau_2 \overline{L}_2]_F + i h_M [L_1^T \tau_2 \Delta_1 L_1 + \overline{L}_2^T \tau_2 \overline{\Delta}_2 \overline{L}_2]_F \quad (4)$$

Renormalization group equations constitute our basic tool in studying the evolution of the

<i>Superfields</i>	$SU(3)_c \times SU(2)_L \times U(1)_Y$	<i>Anomalous Dimension</i>
L	$(1, 2, -\frac{1}{2}\sqrt{\frac{3}{5}})$	$\frac{1}{16\pi^2} [h_\tau^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_Y^2]$
\overline{E}	$(1, 1, \sqrt{\frac{3}{5}})$	$\frac{1}{16\pi^2} [2h_\tau^2 - \frac{6}{5}g_Y^2]$
\overline{D}	$(\overline{3}, 1, \frac{1}{3}\sqrt{\frac{3}{5}})$	$\frac{1}{16\pi^2} [2h_b^2 - \frac{8}{3}g_c^2 - \frac{4}{30}g_Y^2]$
\overline{U}	$(\overline{3}, 1, -\frac{2}{3}\sqrt{\frac{3}{5}})$	$\frac{1}{16\pi^2} [2h_t^2 - \frac{8}{3}g_3^2 - \frac{8}{15}g_Y^2]$
Q	$(3, 2, \frac{1}{6}\sqrt{\frac{3}{5}})$	$\frac{1}{16\pi^2} [h_t^2 + h_b^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_Y^2]$
H_1	$(1, 2, -\frac{1}{2}\sqrt{\frac{3}{5}})$	$\frac{1}{16\pi^2} [h_\tau^2 + 3h_b^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_Y^2]$
H_2	$(1, 2, \frac{1}{2}\sqrt{\frac{3}{5}})$	$\frac{1}{16\pi^2} [3h_t^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_Y^2]$

Table 2: Superfields in MSSM: Representations and anomalous dimensions

relevant Yukawa coupling. Given a trilinear term $d_{abc} \Phi^a \Phi^b \Phi^c$ in the superpotential and the evolution scale μ , the RGE for d_{abc} is [11]

$$\mu \frac{\partial}{\partial \mu} d_{abc} = \gamma_a^i d_{ibc} + \gamma_b^j d_{ajc} + \gamma_c^k d_{abk} \quad (5)$$

In Eqn. 5, γ_a^i is the anomalous dimension matrix

$$\gamma_a^i = Z_a^{-1/2k} \mu \frac{\partial}{\partial \mu} Z_k^{1/2i} = \frac{1}{16\pi^2} [n_p d^2 - 2\delta_a^i \sum_k g_k^2 c_A^k] \quad (6)$$

Where, Z is the renormalization constant matrix relating the renormalized superfield Φ to the unrenormalized superfield Φ_0 by the relation,

$$\Phi_0^i = Z_a^{1/2i} \Phi^a \quad (7)$$

n_p is a numerical factors denoting the number of possible graphs and in usual notation,

$$[\sum_i T^i T^i]_{mn} = c_A \delta_{mn}.$$

We shall apply Eqn. 5 to the Yukawa couplings of interest. First we use this equation and the informations tabulated in Table. 1 and Table. 2 to get the evolution equations of the Yukawa couplings in the MSSM and the MSUSYLR model. Defining $\alpha_i = \frac{g_i^2}{4\pi}$ and $Y_i = \frac{h_i^2}{4\pi}$, we can write those equations. In the region above M_R we have;

$$\begin{aligned} \frac{\partial Y_Q}{\partial t} &= [7Y_Q + Y_l - \frac{16}{3}\alpha_3 - 3\alpha_{2L} - 3\alpha_{2R} - \frac{1}{6}\alpha_{B-L}]Y_Q, \\ \frac{\partial Y_l}{\partial t} &= [3Y_Q + 5Y_l + 4Y_M - 3\alpha_{2L} - 3\alpha_{2R} - \frac{3}{2}\alpha_{B-L}]Y_l, \\ \frac{\partial Y_M}{\partial t} &= [4Y_l + 5Y_M - 7\alpha_{2L} - \frac{9}{2}\alpha_{B-L}]Y_M, \end{aligned}$$

$$\begin{aligned}
\frac{\partial \alpha_3}{\partial t} &= [-9 + 6] \alpha_3^2, \\
\frac{\partial \alpha_{2L}}{\partial t} &= [-6 + 6 + 3] \alpha_{2L}^2, \\
\frac{\partial \alpha_{2R}}{\partial t} &= [-6 + 6 + 3] \alpha_{2R}^2, \\
\frac{\partial \alpha_{B-L}}{\partial t} &= [-0 + 6 + 9] \alpha_{B-L}^2,
\end{aligned} \tag{8}$$

while, in the region $M_{SUSY} < \mu < M_R$ we have,

$$\begin{aligned}
\frac{\partial Y_t}{\partial t} &= [6Y_t + Y_b - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_Y]Y_t, \\
\frac{\partial Y_b}{\partial t} &= [6Y_b + Y_t + Y_\tau - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{15}\alpha_Y]Y_b, \\
\frac{\partial Y_\tau}{\partial t} &= [4Y_\tau + 3Y_b - 3\alpha_2 - \frac{9}{5}\alpha_Y]Y_\tau, \\
\frac{\partial \alpha_3}{\partial t} &= [-9 + 6] \alpha_3^2, \\
\frac{\partial \alpha_{2l}}{\partial t} &= [-6 + 6 + 1] \alpha_{2L}^2, \\
\frac{\partial \alpha_{1Y}}{\partial t} &= [-0 + 6 + \frac{3}{5}] \alpha_{1Y}^2.
\end{aligned} \tag{9}$$

In Eqn. 8 and Eqn. 9 we have used $t = \frac{1}{2\pi} (\ln \frac{\mu}{1GeV})$. For the gauge coupling evolutions the 1 loop beta functions are very well known. For completeness we give the generic formula for the beta function. In our notation we have,

$$\beta = [-3 N + 2 n_f + T_s]$$

where, the first term comes from the gauge contribution and the second term comes from the fermionic contribution. The variable N refers to the gauge group $SU(N)$ and n_f signifies the number of fermionic generations. The last term, T_s , represents the contribution of the scalars.

In our calculation we have assumed $M_{SUSY} = 1TeV$ as such a scale may solve the naturalness problem in the Higgs sector. From the electroweak scale M_Z to the supersymmetry breaking scale M_{SUSY} the evolution of the couplings are governed by non-supersymmetric renormalization group equations [12]

In brief, we have adopted the following procedure. We assume that MSUSYLR symmetry holds good up to a large cut-off scale $M_X = 10^{19}$ GeV. We do not require the gauge couplings to unify. On the other hand, all third generation Yukawa couplings of the MSUSYLR model have been taken to be of order one at M_X as exhibited in Eqn. 10. This scenario of Yukawa couplings in the ultraviolet region is a predictive one, as shown by Hill [2], in the sense that $h_t(m_t)$ is insensitive to the variation of $h_t(M_X)$ within this region. In a more mathematical terminology, $h_t(M_X)$ stays in the domain of attraction. We also think that such an assumption is justified as the third generation fermions are much heavier compared to the rest. The renormalization group equations are solved numerically with the inputs;

$$\frac{h_i^2(M_X)}{4\pi} = 1, \quad \alpha_3(M_Z) = 0.11 - 0.12, \quad \alpha_{2L}(M_Z) = 0.03322, \quad \alpha_{1Y}(M_Z) = 0.01688. \quad (10)$$

We see that as μ decreases the couplings evolve downwards very fast and reach a fixed point at the infrared region. We have varied the right handed symmetry breaking scale M_R in a very wide range; from the TeV region to 10^{18} GeV. The value of the top quark, the bottom quark and the tau lepton Yukawa couplings have been calculated at the scale $M_t = 170$ GeV [$h_t(M_t)$, $h_b(M_t)$ and $h_\tau(M_t)$] with respect to this wide range of variation in the right handed symmetry breaking scale. These results are summarized in the Table 3 and Table 4. The value of the Majorana Yukawa coupling at the right handed symmetry breaking scale [$h_M(M_R)$] also emerges from this analysis. These values are also tabulated in Table 3 and Table 4. Note that below the scale M_R we have considered only the MSSM couplings whereas in general one has a light left handed triplet below M_R . This triplet, when present below M_R , will not have any tree level coupling with the top quark which ensures that our results will be very nearly valid even in that case. The scenario with a low energy triplet and its phenomenological consequences will be studied elsewhere [13].

M_R	$h_t(M_t)$	$h_b(M_t)$	$h_\tau(M_t)$	$h_M(M_R)$	$\tan\beta$	$m_b(M_t)$	$m_t(M_t)$	$\frac{m_b(M_t)}{m_\tau(M_t)}$
10^3	1.02	1.02	0.33	1.16	32.71	5.45	178.7	2.71
10^4	1.01	1.01	0.34	1.17	33.67	5.22	176.7	2.61
10^6	0.99	0.98	0.36	1.20	35.84	4.80	173.7	2.43
10^8	0.98	0.97	0.39	1.24	38.30	4.42	171.8	2.29
10^{10}	0.98	0.96	0.42	1.30	41.13	4.06	171.6	2.11
10^{12}	0.97	0.95	0.45	1.39	44.41	3.72	170.0	1.96
10^{14}	0.97	0.94	0.49	1.55	48.33	3.39	169.8	1.87
10^{16}	0.97	0.93	0.54	1.90	53.25	3.04	170.1	1.65

Table 3: The values of $h_t(M_t)$, $h_b(M_t)$, $h_\tau(M_t)$ and $h_M(M_R)$ calculated by RGE up to the second decimal place for $\alpha_s = 0.11$. The prediction of the masses m_b and m_t defined by the Eqn. 12 and Eqn. 13 at the scale M_t has been quoted in GeV. M_t is defined as 170 GeV.

The value of $\tan\beta$, defined as $\frac{\langle H_2 \rangle}{\langle H_1 \rangle}$, can be estimated from the measured value [14] of the tau lepton mass, having little experimental error, by the equation,

$$m_\tau(m_\tau) \simeq m_\tau(M_t) = h_\tau(M_t) 174 \cos\beta = 1.777 \text{ GeV}, \quad (11)$$

where, H_2 and H_1 are the two Higgs doublets embedded in the Higgs ϕ of MSUSYLR. The estimated value of $\tan\beta$ is tabulated in the sixth columns of Table 3 and Table 4. Once the value of $\tan\beta$ is known the predictions for the top mass and the bottom ¹ mass follows from the equations,

¹ I thank referee for important comments on the prediction of m_b .

M_R	$h_t(M_t)$	$h_b(M_t)$	$h_\tau(M_t)$	$h_M(M_R)$	$\tan\beta$	$m_b(M_t)$	$m_t(M_t)$	$\frac{m_b(M_t)}{m_\tau(M_t)}$
10^3	1.05	1.04	0.33	1.16	32.33	5.63	182.6	3.17
10^4	1.03	1.03	0.34	1.17	33.31	5.39	180.5	3.03
10^6	1.02	1.01	0.36	1.20	35.46	4.96	177.5	2.79
10^8	1.00	0.99	0.38	1.24	37.91	4.56	175.6	2.57
10^{10}	1.00	0.98	0.41	1.30	40.71	4.20	174.4	2.36
10^{12}	1.00	0.97	0.44	1.39	43.98	3.85	173.8	2.16
10^{14}	0.99	0.96	0.48	1.55	47.87	3.50	173.7	1.97
10^{16}	0.99	0.95	0.53	1.90	52.77	3.15	173.9	1.77
10^{17}	1.00	0.95	0.57	2.36	55.91	2.95	174.2	1.66

Table 4: The values of $h_t(M_t)$, $h_b(M_t)$, $h_\tau(M_t)$ and $h_M(M_R)$ calculated by RGE up to the second decimal place for $\alpha_s = 0.12$. The prediction of the masses m_b and m_t defined by the Eqn. 12 and Eqn. 13 at the scale M_t has been quoted in GeV. M_t is defined as 170 GeV.

$$m_b(M_t) = h_b(M_t) 174 \cos\beta, \quad (12)$$

$$m_t(M_t) = h_t(M_t) 174 \sin\beta, \quad (13)$$

which are tabulated in the seventh and eighth columns of Table 3. The prediction for the pole mass of the top quark immediately follows from the equation,

$$m_t(\text{pole}) = m_t(M_t) \left[1 + \frac{4}{3\pi} \alpha_s(M_t) + O(\alpha_s^2) \right]. \quad (14)$$

At this stage we consider the last column of Table 3 and Table 4. The ratio can be estimated from the experimental numbers by the relation,

$$\frac{m_b(M_t)}{m_\tau(M_t)} = \frac{m_b(m_b)}{m_\tau(m_\tau)} \frac{\eta_\tau}{\eta_b}, \quad (15)$$

where η_τ and η_b parametrize [6] the evolution of the masses from their respective scales to the scale M_t . We take the value of $\frac{\eta_\tau}{\eta_b}$ to be 0.74 for $\alpha_s=0.11$ and 0.67 for $\alpha_s=0.12$. Allowing the value of $m_b(m_b)$ in the interval of 4.1 GeV to 4.5 GeV [14] we get the range,

$$1.6 < \frac{m_b(M_t)}{m_\tau(M_t)} < 1.9 \quad \text{for } \alpha_s = 0.11 - 0.12. \quad (16)$$

Now, we can read off a bound on the mass scale M_R from the ninth columns of Table 3 and Table 4,

$$10^{12} \text{ GeV} < M_R < 10^{17} \text{ GeV} \quad \text{for } \alpha_s = 0.11 - 0.12. \quad (17)$$

We have taken a conservative lower bound on M_R realizing that $m_b(m_b)$ may be somewhat larger than 4.5 GeV as well [15]. Now, combining the results in the eighth columns of Table 3 and Table 4 and in Eqn. 17 together with Eqn. 14 we get the one-loop prediction of the physical top mass.

$$177.9 < m_t(\text{pole}) < 183.2 \text{ GeV} \quad \text{for} \quad \alpha_s = 0.11 - 0.12 \quad (18)$$

It is interesting that this scenario of SUSYLR model can predict the top mass in the expected range [1] and also satisfy the measured values of $m_b(m_b)$ and $m_\tau(m_\tau)$.

The lower bound of M_R displayed in Eqn. 17 calls for some comments. There are hints of such a large value of M_R from gauge coupling unification. Our study shows that even independent of the unification of gauge couplings, to predict the correct range of values of the low energy ratio of $\frac{m_b}{m_\tau}$, we need a large value for the scale M_R . We consider it to be a welcome result as a confirmation of the popular MSW [16] solution of the solar neutrino problem will also suggest that M_R is in the range $10^{10} - 10^{12}$ GeV.

To conclude, in this paper we have solved the renormalization group equations of the Yukawa couplings in the SUSYLR model numerically. We have :

- (1) Shown that the fixed point value of the top Yukawa coupling is insensitive to the variation of the right handed symmetry breaking scale. Bound obtained on the top mass is given in Eqn. 18.
- (2) Shown that the low energy ratio of $\frac{m_b}{m_\tau}$ is sensitive to the right handed symmetry breaking scale, which can be used to predict that $10^{12} < M_R < 10^{17}$ GeV.
- (3) Calculated the numerical value of the Majorana Yukawa coupling h_M at the right handed symmetry breaking scale; this coupling is otherwise a free parameter of the SUSYLR model.

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